Electronics Primer for NMR

P. J. Grandinetti*

L'Ohio State Univ.

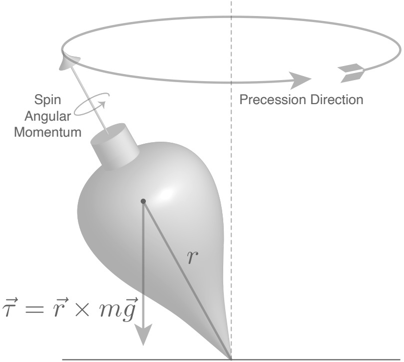
NMR Winter School, 2024

1. Check out Terry Gullion's ENC tutorial video link:
   Basic Useful Circuits for NMR Spectroscopy
2. High Resolution NMR in the Solid State, Stejskal and Memory.

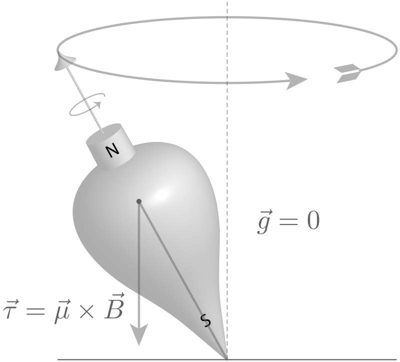
*email: grandinetti.1@osu.edu, web: www.grandinetti.org

Precessing tops

Precessing top



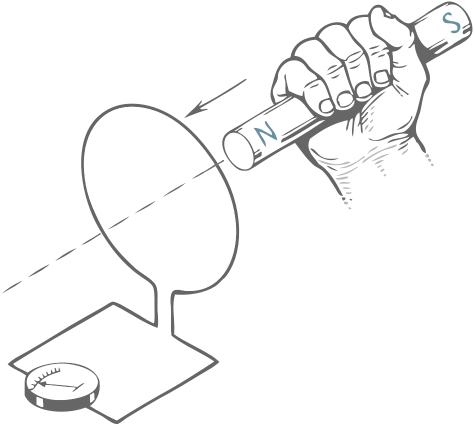
Magnetic top in zero gravity precessing in a magnetic field



How do we measure precession frequency of a magnetic top?

How to measure precession frequency of a magnetic top?

Exploit Faraday's law of induction, discovered in 1831: changing magnetic flux will induce current in surrounding loop of wire.



Faraday's law of induction:

$$\mathcal{E} = -\frac{d\Phi}{dt}$$

$\mathcal{E}$ is EMF and Φ is magnetic flux.

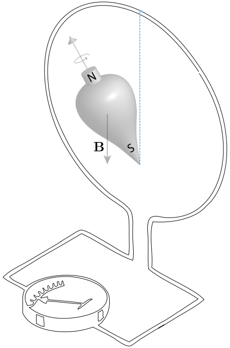
Electromotive Force (EMF, *i.e.*, voltage) induced in coil is related to change in magnetic flux through loop of wire with time.

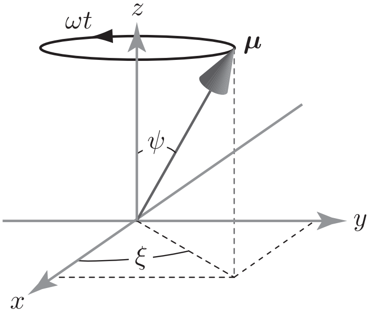
$$\Phi(t) = \int_{\mathcal{S}} \vec{B}(t) \cdot d\vec{a}$$

$\mathcal{S}$ is surface attached to loop of wire.

How to measure precession frequency of a magnetic top?

Place coil of radius R_{coil} around spinning magnetic top to detect precession frequency.





Magnetic dipole vector of top changes with time according to

$$\vec{\mu}(t) = |\vec{\mu}| \left[\sin\psi \cos(\omega t + \xi_0)\ \vec{e}_x + \sin\psi \sin(\omega t + \xi_0)\ \vec{e}_y + \cos\psi\ \vec{e}_z\right]$$

$|\vec{\mu}|$ is length of precessing vector, ψ is angle between precessing vector and z-axis, ξ_0 is initial phase of precessing vector, and ω is angular precession frequency.

Advanced Exercise

1. Show that EMF induced in wire loop surrounding precessing magnetic dipole,

$$\vec{\mu}(t) = |\vec{\mu}|\left[\sin\psi\cos(\omega t + \xi_0)\ \vec{e}_x + \sin\psi\sin(\omega t + \xi_0)\ \vec{e}_y + \cos\psi\ \vec{e}_z\right]$$

is given by

$$\mathcal{E}_x(t) = -\frac{d\Phi_x(t)}{dt} = \omega\frac{\mu_0}{2R_{\text{coil}}}|\vec{\mu}|\sin\psi\sin(\omega t + \xi_0)$$

Hint: Start with definition of magnetic flux and use Stoke's Theorem,

$$\Phi(t) = \int_{\mathcal{S}}\vec{B}_{\rm dip}(t)\cdot d\vec{a} = \int_{\mathcal{S}}(\vec{\nabla}\times\vec{A}_{\rm dip}(t))\cdot d\vec{a} = \oint_{\mathcal{P}}\vec{A}_{\rm dip}(t)\cdot d\vec{l}$$

$\mathcal{P}$ represents circumference of wire loop; $\vec{A}_{\rm dip}(t)$ is magnetic vector potential for point dipole (see Griffiths 3rd Ed. E&M text, p. 244)

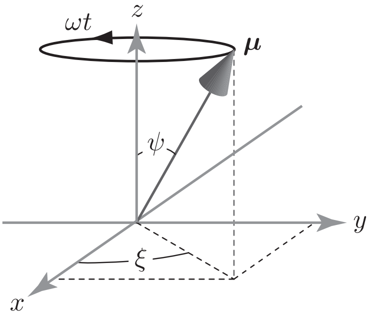
$$\vec{A}_{\rm dip}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{\mu}\times\vec{e}_r}{r^2}.$$

Also, check out "The Principle of Reciprocity," David Hoult, *J. Magn. Reson.*, **213**, 344 (2011).

Faraday Detector and precessing magnetic dipole

$$\mathcal{E}_x(t) = -\frac{d\Phi_x(t)}{dt} = \omega \frac{\mu_0}{2R_{\text{coil}}} |\vec{\mu}| \sin\psi \sin(\omega t + \xi_0)$$

Measure precession frequency from EMF signal, $\mathcal{E}_x(t)$



- Amplitude increases with precession frequency, ω. At higher magnetic field strengths, magnetic top precesses faster and gives larger amplitudes.
- Amplitude increases with decreasing coil radius, R_{coil}.
- Amplitude increases with magnetic dipole moment strength, $|\vec{\mu}|$.
- Amplitude scaled by $\sin\psi$, i.e., amplitude increases as magnetic dipole is tilted further away from z-axis.

Exercises

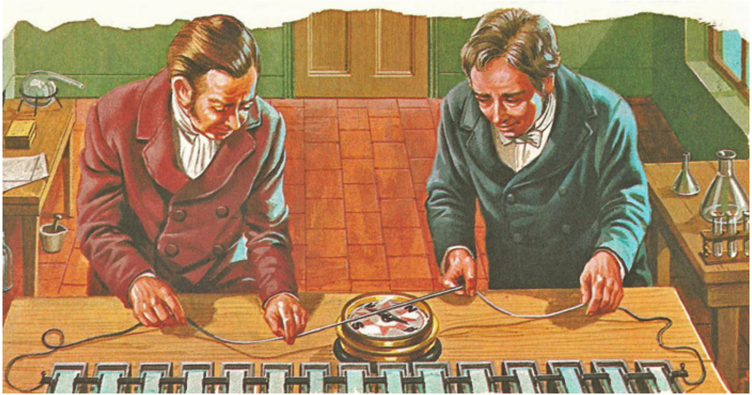
1. Based on

$$\mathcal{E}_x(t) = -\frac{d\Phi_x(t)}{dt} = \omega\frac{\mu_0}{2R_{\text{coil}}}|\vec{\mu}|\sin\psi\sin(\omega t + \xi_0)$$

   calculate the increase in signal from doubling the external magnetic field strength.

2. Explain why no NMR signal is usually detected when the long axis of the NMR receiver coil is parallel to the external magnetic field direction.

In 1820 Hans Christian Ørsted discovered that electric current produces a magnetic field that deflects compass needle from magnetic north, establishing first direct connection between fields of electricity and magnetism.

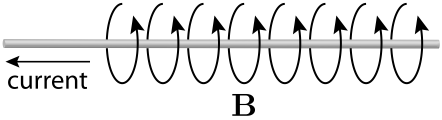


Biot-Savart Law

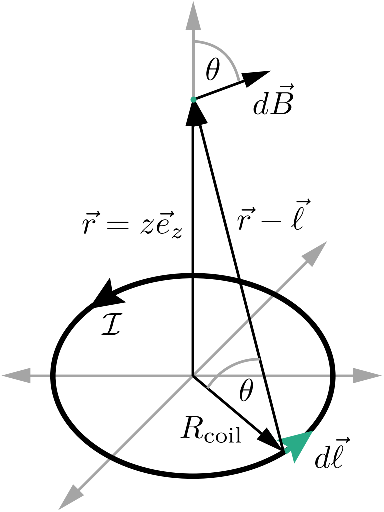
Jean-Baptiste Biot and Félix Savart worked out that magnetic flux density produced at distance r away from section of wire of length dl carrying steady current $\mathcal{I}$ is

$$d\vec{B} = \frac{\mu_0 \mathcal{I}}{4\pi} \frac{d\vec{l} \times \vec{r}}{r^3} \qquad \text{Biot-Savart law}$$

Direction of magnetic field vector is given by "right-hand" rule: if you point thumb of your right hand along direction of current then your fingers will curl in direction of magnetic field.



Calculate magnetic field produced by current in wire loop



- Magnetic field along z axis away from current loop

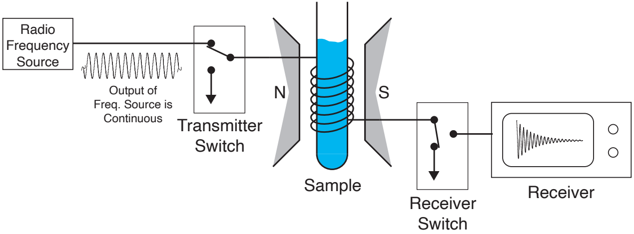
$$\vec{B}(z) = \vec{e}_z \frac{\mu_0 \mathcal{I}}{4\pi} \frac{\cos\theta}{R_{\text{coil}}^2 + z^2} \int d\vec{\ell}$$
$$= \vec{e}_z \frac{\mu_0 \mathcal{I}}{4\pi} \frac{R_{\text{coil}}^2}{(R_{\text{coil}}^2 + z^2)^{3/2}}$$

- Magnetic field at center of current loop

$$\vec{B}(0) = \vec{e}_z \frac{\mu_0 \mathcal{I}}{4\pi} \frac{1}{R_{\text{coil}}}$$

Magnetic field strength at center scaled by inverse of coil radius.

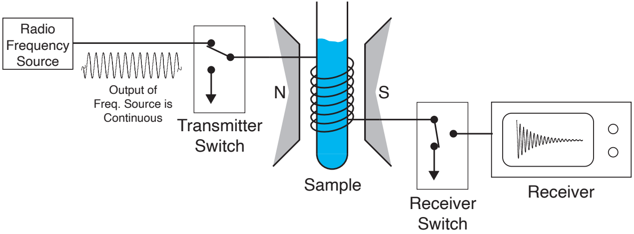
Are you a maker? Let's build an NMR Spectrometer!



6 essential components in our primitive NMR spectrometer:

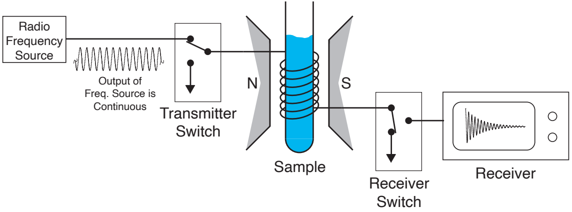
1. a radio frequency (RF) source tuned to resonance frequency of nuclei
2. a switch (or gate) for turning RF irradiation on and off
3. a magnet to polarize and split nuclear spin energy levels
4. a transmitter and detector coil containing sample
5. a switch (or gate) in front of receiver for protection
6. receiver, which could simply be oscilloscope in this design

Are you a maker? Let's build an NMR Spectrometer!



- Construction of such an instrument is straightforward.
- Because time scale of NMR experiment is on order of microseconds, switching times for gates need to be on the order of nanoseconds or less for precise time resolved measurements.
- Computer controlled low power radio frequency gates having such switching speeds are readily available commercially. Check out link: www.minicircuits.com
- In primitive spectrometer computer controls timing for opening and closing of transmitter and receiver gates. Check out link: Arduino boards

A Primitive NMR Spectrometer



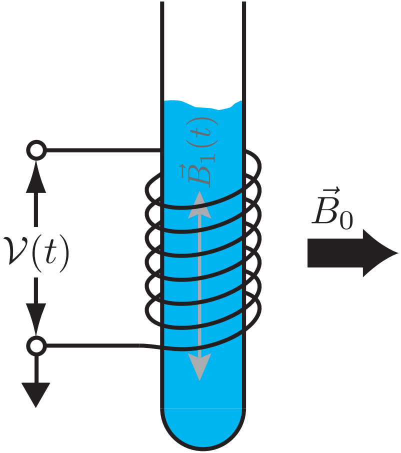
Simplest experiment—pulse and detect signal—consist of 3 steps:

Step	Transmitter switch state	Receiver switch state	Duration
1	OFF	OFF	30 seconds
2	ON	OFF	4 microseconds
3	OFF	ON	100 milliseconds

- Elementary version of "Pulse Sequence" or "Pulse Program"
- Important part of pulse sequence is duration of each step, or event.
- NMR spectrometer computers control more switches than just receiver and transmitter
- NMR spectrometers have pulse sequence languages for controlling devices in loops, with if statements, and other possibilities.

The NMR Probe

In spectrometer coil wrapped around sample is called "transceiver coil".



1. Used to produce oscillating B_1 field that rotates magnetization
2. Used as Faraday detector of precessing magnetization after pulse

- Let's examine what is needed to enhance efficiency of this coil in producing B_1 fields.
- These same changes, in turn, will also enhance efficiency of this coil as a detector.

Begin by reviewing basics about oscillating voltages and currents in electronic components like resistors, capacitors, and inductors.

The Resistor

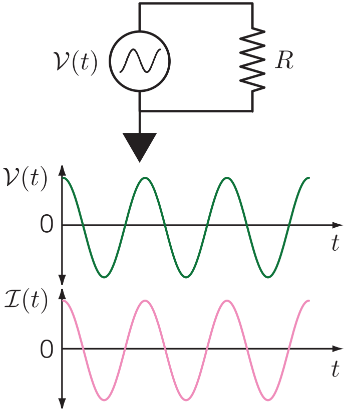
- Apply oscillating voltage across resistor, R

$$\mathcal{V}(t) = \mathcal{V}_0 \cos \omega t$$

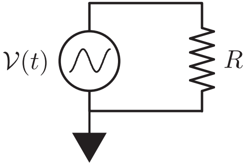
- Oscillating current across resistor is

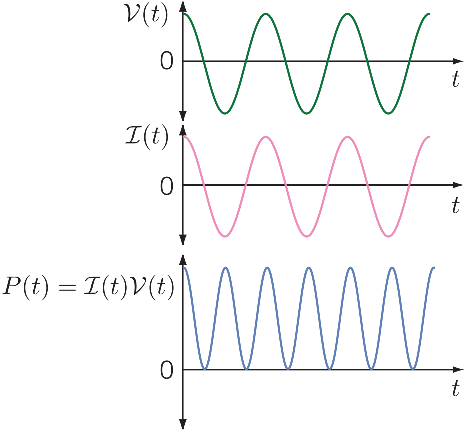
$$\mathcal{I}(t) = \frac{\mathcal{V}(t)}{R} = \frac{\mathcal{V}_0}{R} \cos \omega t = \mathcal{I}_0 \cos \omega t$$

- Maximum current across resistor is $\mathcal{I}_0 = \dfrac{\mathcal{V}_0}{R}$



Instantaneous Power in Purely Resistive Circuits





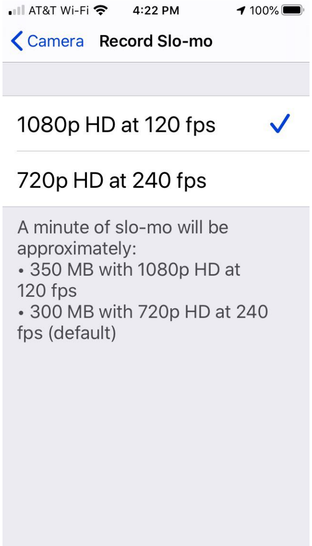
$$\mathcal{V}(t) = \mathcal{V}_0 \cos \omega t \quad \text{and} \quad \mathcal{I}(t) = \mathcal{I}_0 \cos \omega t$$

Calculate power $P(t) = \mathcal{I}(t)\mathcal{V}(t)$

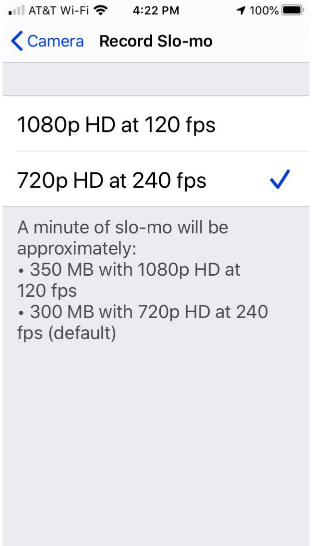
$$P(t) = \mathcal{I}_0\mathcal{V}_0 \cos^2 \omega t = \frac{1}{2}\mathcal{I}_0\mathcal{V}_0 - \frac{1}{2}\mathcal{I}_0\mathcal{V}_0 \cos 2\omega t$$

- Integrate $P(t)$ over one cycle and find average power consumed in one cycle is $\frac{1}{2}\mathcal{I}_0\mathcal{V}_0$.
- 60 Hz AC leads to 120 Hz light flickering

Instantaneous Power in Purely Resistive Circuits



Instantaneous Power in Purely Resistive Circuits



The Inductor

- Apply oscillating voltage across inductor, L

$$\mathcal{V}(t) = \mathcal{V}_0 \cos \omega t$$

- Oscillating current across inductor is

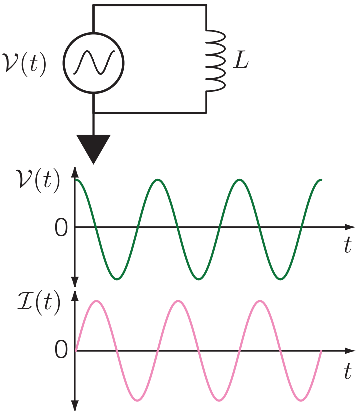
$$\mathcal{I}(t) = \int_{t_0}^{t} \frac{\mathcal{V}(t)}{L} ds = \frac{\mathcal{V}_0}{\omega L} \sin \omega t = \mathcal{I}_0 \sin \omega t$$

- Current is -90° out of phase with voltage oscillation.

$$\mathcal{I}(t) = \frac{\mathcal{V}_0}{\omega L} \cos(\omega t - \pi/2) = \mathcal{I}_0 \cos(\omega t - \pi/2)$$

In inductor, voltage lags current by 90°.

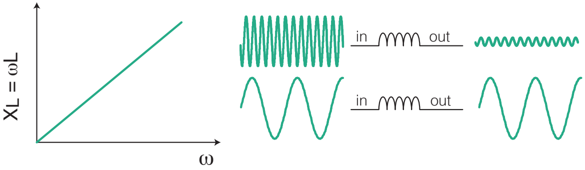
- Maximum current across inductor is $\mathcal{I}_0 = \dfrac{\mathcal{V}_0}{\omega L}$



The Inductor

$$\mathcal{I}_0 = \frac{\mathcal{V}_0}{\omega L}$$

- Inductor behaves like frequency dependent reactance of ωL with current -90° out of phase with voltage oscillation. Because it is 90° out of phase it is called *reactance* (not resistance).
- Inductor blocks (reacts against) high frequency currents, but allows low frequencies to pass.



- Problem with using inductor alone ...
  - as a receiver coil is that the oscillating current amplitude, due to precessing magnetization, is inversely related to precession frequency.
  - as a transmitter coil, is that current (and B_1 field strength) decreases with increasing frequency, i.e., need higher voltages to push same current through coil to get same B_1 field at higher frequencies.

The Inductor - Practical Note

Estimate inductance of coil from its dimensions. Inductance, L, in μH is

$$L = \frac{r^2 n^2}{9r + 10l}$$

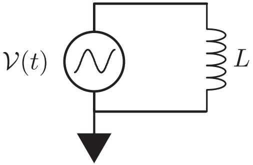
- r is radius of coil in inches,
- n is number of turns,
- l is coil length in inches.

Quality factor or Q factor of inductor at operating frequency ω is defined as ratio of reactance of coil to its intrinsic resistance, R_t,

$$Q = \frac{\omega L}{R_t}$$

Optimum Q is attained when the length of the coil (l) is equal to its diameter $(2r)$

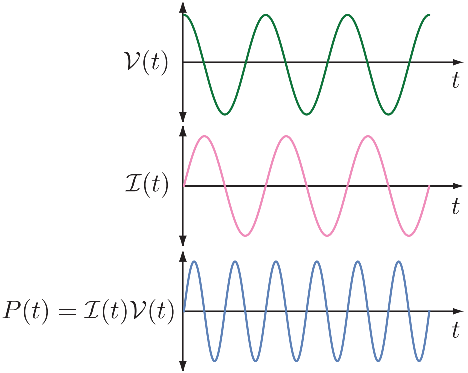
Instantaneous Power in Purely Inductive Circuits



$$\mathcal{V}(t) = \mathcal{V}_0 \cos\omega t \quad \text{and} \quad \mathcal{I}(t) = \mathcal{I}_0 \cos(\omega t - \pi/2) = \mathcal{I}_0 \sin(\omega t)$$

$$P(t) = \mathcal{I}(t)\mathcal{V}(t) = \mathcal{I}_0\mathcal{V}_0 \cos\omega t \sin\omega t = \frac{1}{2}\mathcal{I}_0\mathcal{V}_0 \sin 2\omega t$$

- Average power consumed over one cycle is zero.
- Pure inductive circuit never consumes power.
- When power is positive, energy gets stored in magnetic field created by current in inductor. When power is negative, this energy is returned back to the supply.



The Capacitor

- Apply oscillating voltage across capacitor, C

$$\mathcal{V}(t) = \mathcal{V}_0 \cos \omega t$$

- Oscillating current across inductor is given by

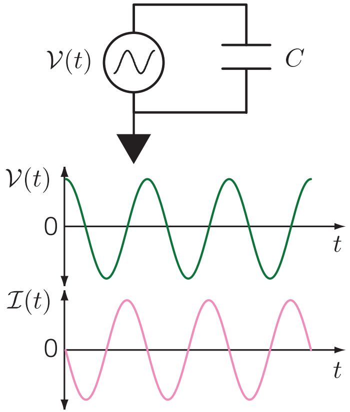
$$\mathcal{I}(t) = C\,\frac{d\mathcal{V}(t)}{dt} = -C\omega\mathcal{V}_0 \sin \omega t = \mathcal{I}_0 \sin \omega t$$

- Current is 90° out of phase with voltage oscillation

$$\mathcal{I}(t) = C\omega\mathcal{V}_0 \cos(\omega t + \pi/2) = \mathcal{I}_0 \cos(\omega t + \pi/2)$$

  In capacitor, voltage leads current by 90°.

- Maximum current across capacitor is $\mathcal{I}_0 = \dfrac{\mathcal{V}_0}{1/(\omega C)}$

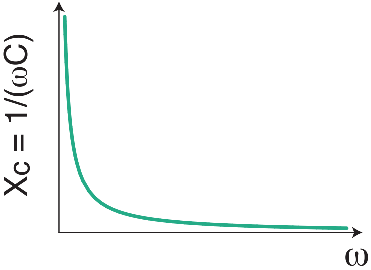


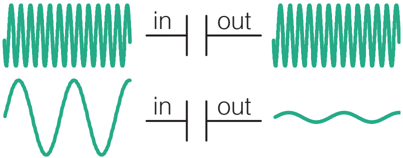
The Capacitor

$$\mathcal{I}_0 = \frac{\mathcal{V}_0}{1/(\omega C)}$$

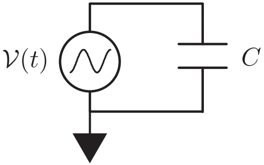
Capacitor behaves like frequency dependent reactance of $1/(\omega C)$ with current 90° out of phase with voltage oscillation.

Capacitor blocks (reacts against) low frequency currents, but allows high frequencies to pass—the opposite behavior of inductor.



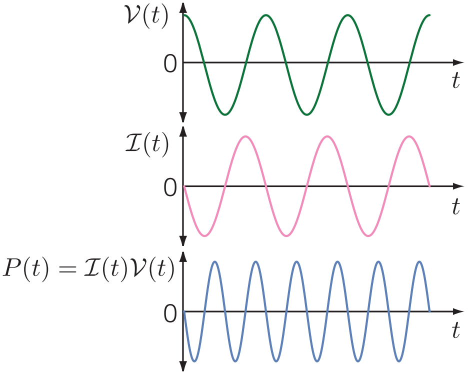


Instantaneous Power in Purely Capactive Circuits



$$\mathcal{V}(t) = \mathcal{V}_0 \cos \omega t \quad \text{and} \quad \mathcal{I}(t) = \mathcal{I}_0 \cos(\omega t + \pi/2) = -\mathcal{I}_0 \sin(\omega t)$$

$$P(t) = \mathcal{I}(t)\mathcal{V}(t) = -\mathcal{I}_0 \mathcal{V}_0 \cos \omega t \sin \omega t = -\frac{1}{2}\mathcal{I}_0 \mathcal{V}_0 \sin 2\omega t$$



- Average power consumed over one cycle is zero.
- Pure capacitive circuit never consumes power.
- When power is positive, energy gets stored in electric field of capacitor. When power is negative, this energy is returned back to the supply.

Complex Voltage and Current

Relationships between maximum current and maximum voltage across a resistor, capacitor, and inductor

$$\mathcal{I}_0 = \frac{\mathcal{V}_0}{R} \quad \text{(resistor)}, \qquad \mathcal{I}_0 = \frac{\mathcal{V}_0}{1/(\omega C)} \quad \text{(capacitor)}, \quad \text{and} \qquad \mathcal{I}_0 = \frac{\mathcal{V}_0}{\omega L} \quad \text{(inductor)}$$

Need amplitude and phase relationship for current and voltage, not just maximum values.
Defining complex voltage $\mathcal{V}_c(t) = \mathcal{V}_0 e^{i\omega t}$, where actual voltage is real part,

$$\mathcal{V}(t) = \Re\{\mathcal{V}_0 e^{i\omega t}\} = \mathcal{V}_0 \cos \omega t$$

Similarly, define complex current $\mathcal{I}_c(t) = \mathcal{I}_0 e^{i\omega t}$, where actual current is real part

$$\mathcal{I}(t) = \Re\{\mathcal{I}_0 e^{i\omega t}\} = \mathcal{I}_0 \cos \omega t$$

To phase shift $\mathcal{I}(t)$ by 90° multiply complex voltage $\mathcal{I}_c(t)$ by $e^{i\pi/2}$, and take real part:

$$\mathcal{I}(t) = \Re\{\mathcal{I}_0 e^{i\omega t} e^{i\pi/2}\} = \mathcal{I}_0 \cos(\omega t + \pi/2)$$

Capacitor Impedance

Applying this approach to current-voltage relationship for capacitors we write

$$\mathcal{I}(t) = \frac{\mathcal{V}_0}{1/(\omega C)} \cos(\omega t + \pi/2) = \Re\left\{ \frac{\mathcal{V}_c(t) e^{i\pi/2}}{1/(\omega C)} \right\}$$

Since $e^{i\pi/2} = i$ we can simplify to

$$\mathcal{I}(t) = \Re\left\{ \frac{\mathcal{V}_c(t)}{-i/(\omega C)} \right\} = \Re\left\{ \frac{\mathcal{V}_c(t)}{Z_C} \right\}$$

where $Z_C = -i/(\omega C)$ is the *impedance of the capacitor*.
In terms of complex current across capacitor we write

$$\mathcal{I}_c(t) = \frac{\mathcal{V}_c(t)}{Z_C}$$

Obtain familiar Ohm's law, describing relationship between current and voltage at all times, but now it includes phase information.

Inductor Impedance

Applying this approach to current-voltage relationship for inductors we find

$$\mathcal{I}(t) = \frac{\mathcal{V}_0}{\omega L} \cos(\omega t - \pi/2) = \Re\left\{\frac{\mathcal{V}_c(t)}{i\omega L}\right\}$$

In terms of complex current across inductor we write

$$\mathcal{I}_c(t) = \frac{\mathcal{V}_c(t)}{Z_L}$$

where $Z_L = i\omega L$ is the *impedance of the inductor*.

Impedance, Z

$$Z = R + iX$$

R is **Resistance**: impedes current flow from collisional processes and dissipates energy as heat. Analogous to friction.

X is **Reactance**: impedes current from changing electric and magnetic fields associated with alternating currents. It is not associated with power dissipation. Analogous to inertia.

$Z_R = R$ for resistors, $Z_C = -i/(\omega C)$ for capacitors, and $Z_L = i\omega L$ for inductors

Ohm's law can be generalized to include inductors and capacitors.

For components in series we have

$$Z_T = Z_1 + Z_2 + Z_3 + \cdots$$

For components in parallel we have

$$1/Z_T = 1/Z_1 + 1/Z_2 + 1/Z_3 + \cdots$$

Average power dissipated in the nuclei

Coil inductance, L, increases when filled with material with magnetic susceptibility, χ_0,

$$L = L_0 \left[1 + 4\pi\chi_0\right]$$

affecting amplitude and phase of magnetic field inside coil.
Interested in frequency dependence of magnetic susceptibility, which can written as a complex quantity

$$\chi(\omega) = |\chi(\omega)|e^{-i\omega t} = \chi'(\omega) - i\chi''(\omega)$$

The impedance of sample coil inductor is

$$\begin{aligned} Z &= i\omega L_0 \left[1 + 4\pi\chi(\omega)\right] + R_t \\ &= i\omega L_0 \left[1 + 4\pi\left(\chi'(\omega) - i\chi''(\omega)\right)\right] + R_t \\ &= i\omega L_0 \left[1 + 4\pi\chi'(\omega)\right] + \underbrace{\omega L_0 \chi''(\omega)}_{R_n} + R_t \end{aligned}$$

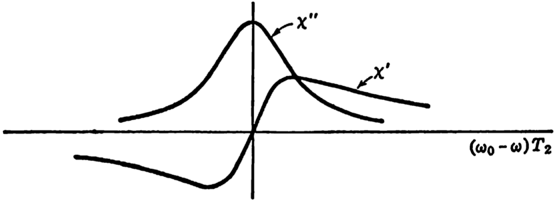


Fig. 2.12. χ' and χ'' from the Bloch equations plotted versus $x \equiv (\omega_0 - \omega)T_2$

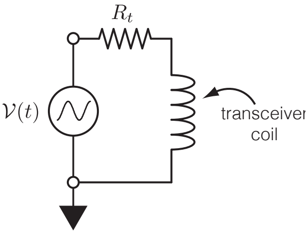
C.P.Slichter, *Principles of Magnetic Resonance*, Chapter 2.

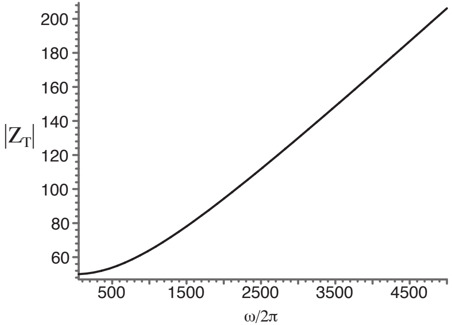
Average power dissipated in the nuclei

$$\overline{P} = \frac{1}{2} I^2 R_n = \frac{1}{2} I^2 \omega L_0 \chi''(\omega)$$

A Tuned Circuit

Ready to solve our problem with transceiver coil. To make problem more realistic include wire resistance, R_t, as in circuit below.





Total impedance is

$$Z_T = Z_R + Z_L = R_t + i\omega L$$

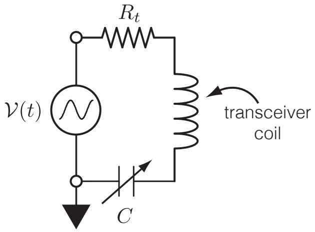
Magnitude of impedance is

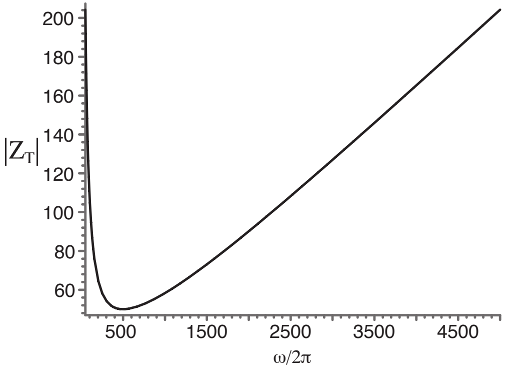
$$|Z_T| = \sqrt{Z_T Z_T^*} = \sqrt{R_t^2 + \omega^2 L^2}$$

- Lowest impedance and highest current only at $\omega = 0$
- Signal oscillates at MHz frequencies so not optimal.

A Tuned Circuit

We can solve this problem by adding a capacitor in series as shown below.



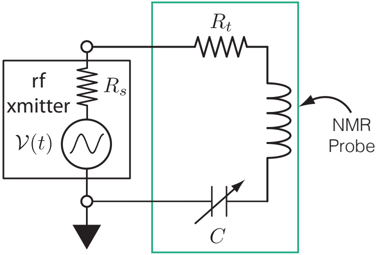


Total impedance is

$$Z_T = Z_R + Z_L + Z_C = R + i\left(\omega L - \frac{1}{\omega C}\right)$$

At $\omega_0 = 1/\sqrt{LC}$ then $Z_L + Z_C = 0$ and $Z_T = R$
Highest current amplitude at ω_0.

A Tuned and Matched Circuit



$$Z_T = R_s + R_t + i\left(\omega L - \frac{1}{\omega C}\right)$$

at resonant frequency $\omega_0 = 1/\sqrt{LC}$

$$Z_T = R_s + R_t$$

Current is $\mathcal{I}(\omega_0) = \dfrac{\mathcal{V}(\omega_0)}{R_s + R_t}$

Power in the coil is

$$P(\omega_0) = \mathcal{I}^2(\omega_0) R_t = \frac{\mathcal{V}^2(\omega_0) R_t}{(R_s + R_t)^2}$$

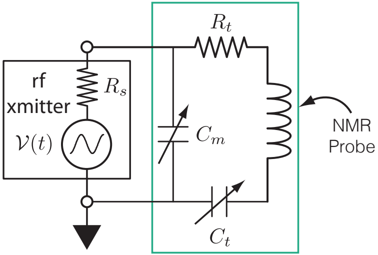
Want maximum power transfer from transmitter to coil.

$$\frac{dP(\omega_0)}{dR_t} = \frac{\mathcal{V}(\omega_0)^2}{(R_s + R_t)^2} - \frac{2\mathcal{V}(\omega_0)^2 R_s}{(R_s + R_t)^3} = 0$$

- Solving this expression gives the condition $R_t = R_s$ for the maximum power transfer.
- When $R_t = R_s$ we say that the impedance of the tuned circuit is "matched" to the transmitter's impedance.

A Tuned and Matched Circuit

- Transmitter's impedance (resistance) is usually fixed at $R_s = 50\ \Omega$.
- R_t is generally less than $1\ \Omega$.
- How do we match impedances? Add another capacitor to circuit.

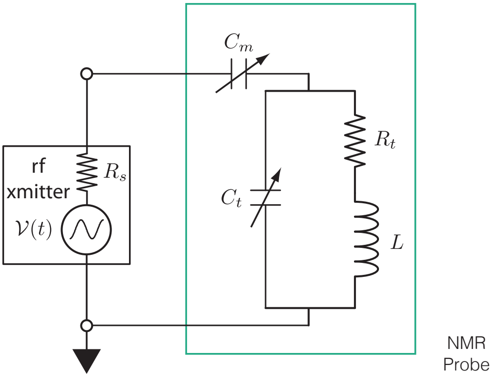


In this series tuned, parallel matched circuit probe impedance is

$$\frac{1}{Z_T} = i\omega C_m + \frac{1}{R_t + i\left(\omega L - \dfrac{1}{\omega C_t}\right)}$$

If C_t and C_m are adjusted so impedance is completely real (no imaginary part) and equal to $Z_T = 50\ \Omega$ at frequency ω_0 then we have maximum power transfer between transmitter and sample, or also between sample and receiver.

Another Tuned and Matched Circuit

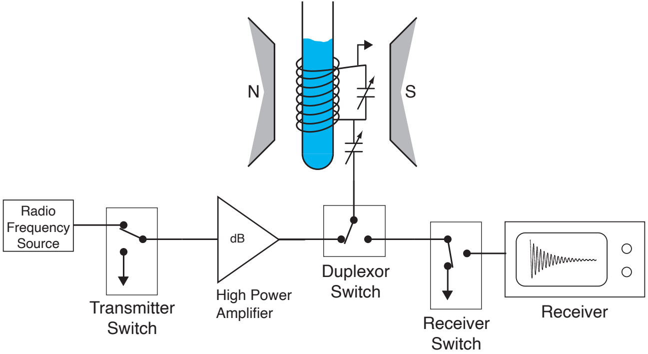


To learn more...

1. Check out Terry Gullion's ENC tutorial video link: Basic Useful Circuits for NMR Spectroscopy
2. High Resolution NMR in the Solid State, Stejskal and Memory.

The Duplexor

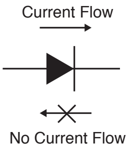
Our spectrometer needs to switch probe between transmitter and receiver to protect receiver from high power RF pulse of transmitter called the *duplexor*.



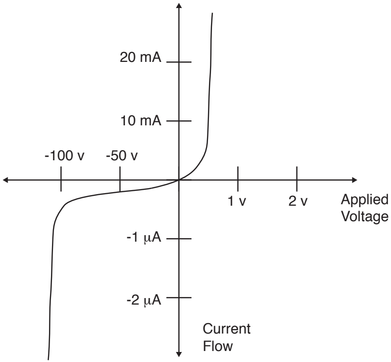
- Because duplexer switches between high power RF from transmitter and low power RF from probe there is an inexpensive passive circuit that can be used to rapidly perform this switching.
- To understand how this works need to review 2 important devices:
  (1) the cross diodes, and (2) quarter-wave transmission lines.

Diode

- Diode symbol: 
- Diodes is one-way valve for current (i.e. direction of arrow).



Plot of current flow versus applied voltage for diode

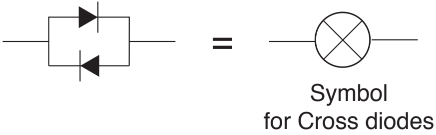


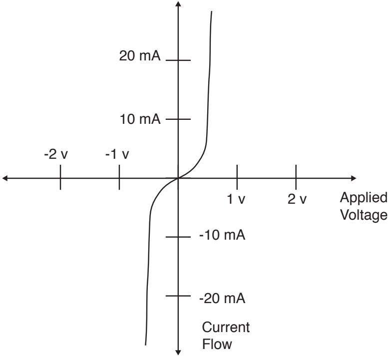
Note axes ranges.
For diode current flows forward after voltage of greater half a volt is applied.

Cross-Diodes

Cross-diodes have property that current flows in either direction as long as voltage is greater than half a volt in magnitude.

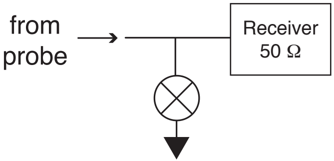
Two diodes connected antiparallel





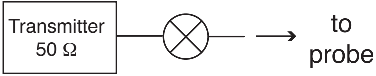
Cross-Diodes in NMR

Use cross-diode to protect receiver from high power RF pulse



High voltage pulse would turn diodes on and go to ground instead of going into receiver with its $50\ \Omega$ impedance.

Use cross-diode to block low power noise from transmitter from entering probe.

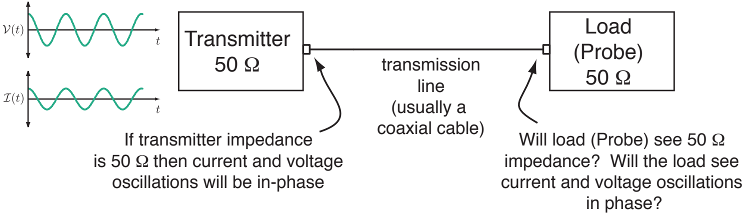


- Broadband noise from transmitter can easily saturate NMR signal and needs to be eliminated.
- As long as noise voltage doesn't exceed threshold voltage of diode it will be blocked from going to probe.

- When signal-to-noise ratio unexplainably drops it is often a blown cross-diode that is problem.
- Solid-state NMR experiments which use long high power pulses such as cross-polarization are often responsible for blown out diodes.

Transmission lines

Impedance of all devices (i.e. probes, transmitter, receiver) need to be matched for maximum power transfer.
Connect all these devices together making sure impedance is $50\ \Omega$ everywhere.



To match impedance of source, Z_s, and load, Z_l, the characteristic impedance of transmission line, Z_0, must be

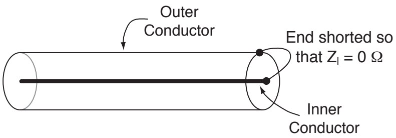
$$Z_0 = \sqrt{Z_s Z_l} = \sqrt{50 \cdot 50} = 50\ \Omega$$

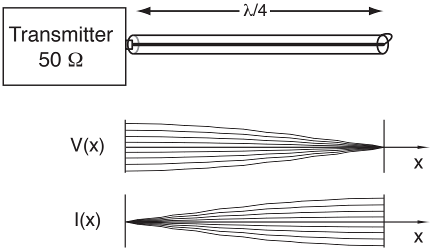
If transmission line is terminated by load that doesn't match its characteristic impedance then voltage and current waves are partially reflected and standing waves are set up in transmission line.

Quarter-Lambda lines - Shorted End

Terminate transmission line with short to ground, that is, connect inner conductor to outer conductor

All voltage and current oscillation will reflect and set up a standing wave.





Voltage is zero and current is maximum at shorted-to-ground end.

Current is zero and voltage is maximum at $\lambda/4$ away from shorted-to-ground end—where source is connected.

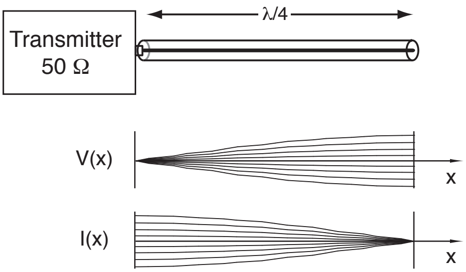
Cable impedance at point where source is connected looks like

$$Z = \frac{\mathcal{V}(0)}{\mathcal{I}(0)} = \frac{\mathcal{V}_0}{0 \text{ amps}} = \infty\ \Omega$$

Transmitter can't tell difference between nothing and $\lambda/4$ with shorted end connected.

Quarter-Lambda lines - Open End

What if we didn't short the end, but left the two conductors unconnected?



Voltage is maximum, and current is zero at open end.
Current is maximum and voltage is zero at $\lambda/4$ away from open end—where source is connected.
Cable impedance at point where source looks like

$$Z = \frac{\mathcal{V}(0)}{\mathcal{I}(0)} = \frac{0 \text{ volts}}{\mathcal{I}_0} = 0\ \Omega$$

All transmitter power is being sent to ground when $\lambda/4$ with an open end is attached.

Quarter-Lambda lines – Summary

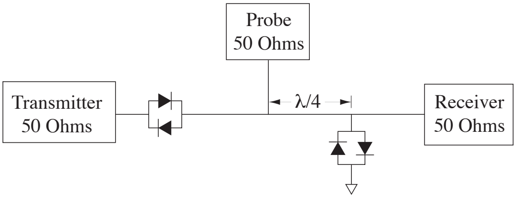
For current–voltage oscillations at a frequency of $\omega = 2\pi/\lambda$ we find that

- $\lambda/4$ length cables with shorted ends look like $Z = \infty$ impedance at the source.
- $\lambda/4$ length cables with open ends look like $Z = 0$ impedance (short to ground!) at the source.

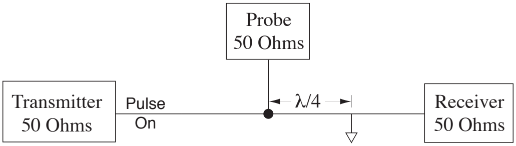
Counter intuitive if you've only ever thought about DC circuits.

Duplexor Circuit

Duplexer switches probe between transmitter and receiver. Consider circuit below:



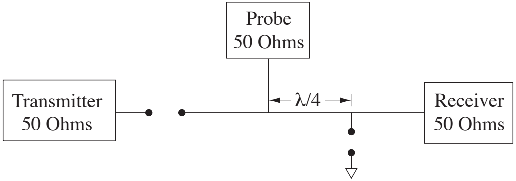
High voltage pulse from transmitter turns on **all** cross-diodes and transmitter sees



At tee (marked with dot) transmitter sees $50\ \Omega$ load of probe, and infinite load in front of receiver. No RF pulse goes into receiver.

Duplexor Circuit

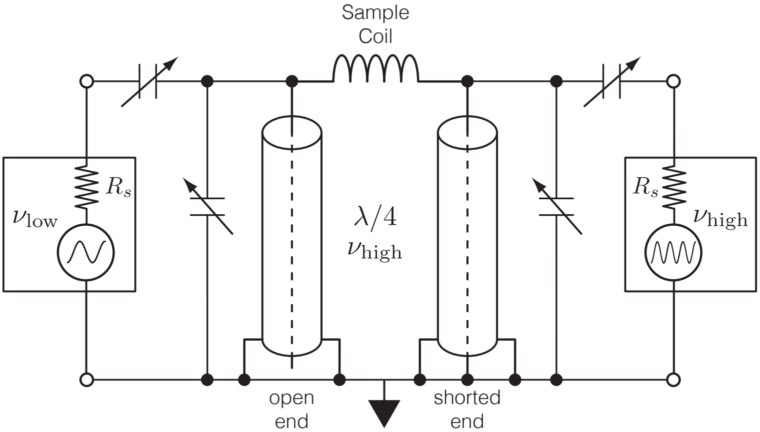
When transmitter is off, then weak NMR signal from probe cannot turn on diodes:



- All cross-diodes are off so probe signal goes only to receiver.
- Cross diode also blocks noise from transmitter from reaching probe.
- Since $\lambda/4$ length depends on frequency (i.e. $\lambda = c/\nu$) then any time you change NMR frequency (i.e. when changing to different nucleus), duplexer has to be changed (different $\lambda/4$ cable).
- With wrong $\lambda/4$ length, then part of transmitter power will go to ground and not to probe.
- If cross-diodes are blown (current flows in both directions with no resistance in blown diodes), then transmitter noise will saturate magnetization and signal from probe will not go only to receiver and sensitivity will suffer.

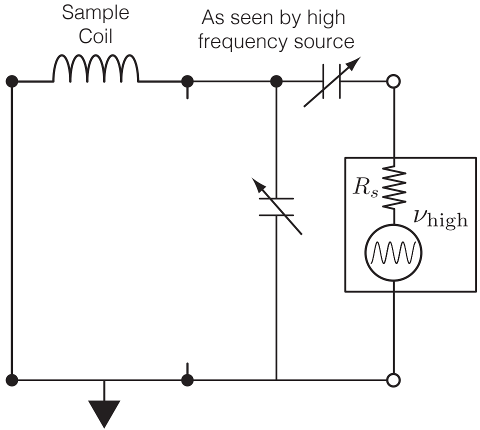
Double Resonance Probe with single coil for both nuclei

Cross, Hester, Waugh



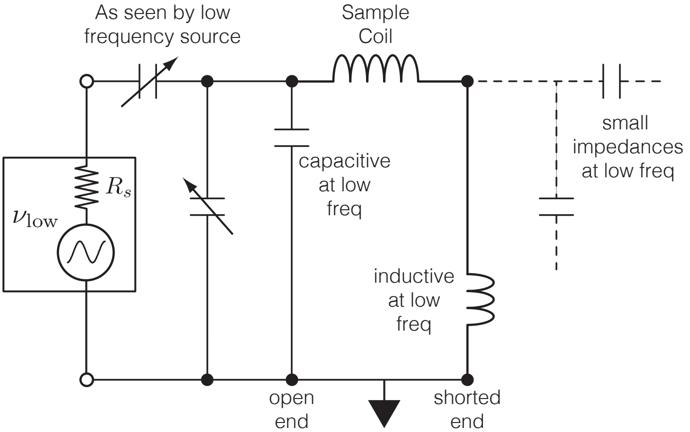
Double Resonance Probe with single coil for both nuclei

Cross, Hester, Waugh



Double Resonance Probe with single coil for both nuclei

Cross, Hester, Waugh



Specifying Power Levels

Power is energy transfer per unit time.

$$P = \frac{\mathcal{V}_{rms}^2}{R} \quad \text{and} \quad \mathcal{V}_{rms} = \frac{\mathcal{V}_{pp}}{2\sqrt{2}}$$

$\mathcal{V}_{rms}$ is rms voltage and $\mathcal{V}_{pp}$ is peak to peak voltage.

Example

Calculate power from $50\ \Omega$ RF source with output of $\mathcal{V}_{pp} = 0.632$ V

$$\mathcal{V}_{rms} = \frac{\mathcal{V}_{pp}}{2\sqrt{2}} = \frac{0.632\text{ V}}{2\sqrt{2}} = 0.2236\text{ V}$$

$$P = \frac{(0.2236\text{ V})^2}{50\Omega} = 0.001\text{ W or 1 mW.}$$

Amplifier Gain

Amplifier gain given in decibels (dB). Logarithmic scale calculated according to

$$dB = 20 \log_{10} \frac{(\mathcal{V}_{pp})_{\text{out}}}{(\mathcal{V}_{pp})_{\text{in}}}$$

If amplifier input is $\mathcal{V}_{pp} = 0.632$ V then after 50 dB gain we get

$$(\mathcal{V}_{pp})_{\text{out}} = (\mathcal{V}_{pp})_{\text{in}} \cdot 10^{dB/20} = (0.632 \text{ volts}) \cdot 10^{50/20} = 200 \text{ V}$$

After 50 dB amplifier, 1 mW would be amplified to

$$P = \frac{(200 \text{ V}/(2\sqrt{2}))^2}{50\ \Omega} = 100 \text{ W}.$$

Checking Power Levels

RF levels are also specified in units of dBm, given by

$$\text{dBm} = 10 \cdot \log \frac{P(\text{mW})}{1\ \text{mW}}$$

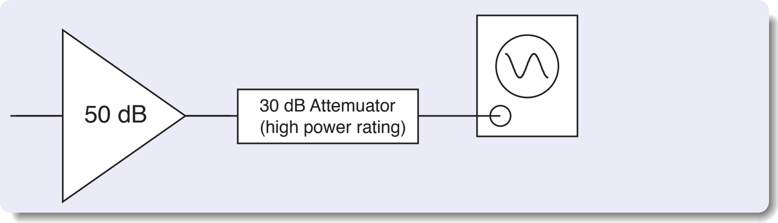
- dBm is gain in terms of dB's with respect to 1 mW.
- If RF source outputs 1 mW, then power in dBm is zero.
- A 50 dB amplifier turns 0 dBm into 50 dBm—which is 100 W.

Amplifiers have a maximum input level.

- Anything higher will overdrive amplifier and lead to distorted output (higher harmonics added).
- Important to check RF power levels going into probe and make sure they are within specifications.
- **Never connect the output of a high power amplifier directly to the oscilloscope.** Oscilloscope may not handle that much power and can be damaged.

Checking Power Levels

Place high power attenuator (check power rating) between amplifier and oscilloscope.

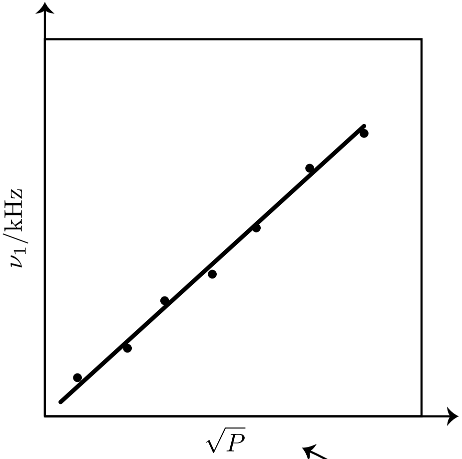


high power
RF attenuator

With setup above

1. measure voltage peak-to-peak on oscilloscope,
2. convert this to dBm,
3. add 30 dB to get output power level of amplifier.

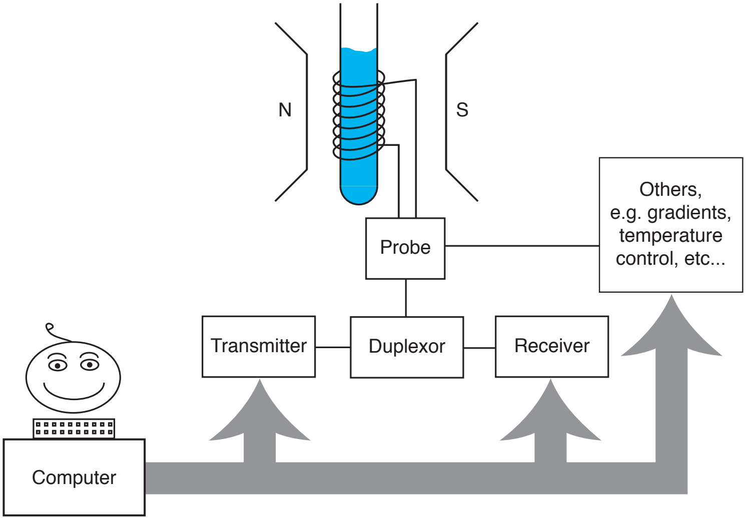
Make Calibration Plot for Probe



Measure NMR signal as a function of pulse length

Measure power going into probe

Let's build an NMR Spectrometer!



Further Reading

- Transient Techniques in NMR of Solids: An Introduction to Theory and Practice, by Gerstein and Dybowski
- Experimental Pulse NMR: A Nuts and Bolts Approach, by Fukushima and Roeder
- The ARRL Handbook for Radio Communication
- Radio-Frequency Electronics, Circuits and Applications, by Hagen
- Practical Electronics for Inventors, by Scherz and Monk
- Practical Exercises for Learning to Construct NMR/MRI Probe Circuits, Conradi and Wheeler, Concepts in Magn. Reson., Part A, **40**A(1) 1–13 (2012).

Again...

1. Check out Terry Gullion's ENC tutorial video link:
   Basic Useful Circuits for NMR Spectroscopy
2. High Resolution NMR in the Solid State, Stejskal and Memory.